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ON SPACE HYPERSONIC FLOWS

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A stationary hypersonic flow past an arbitraty body is considered; both a drag and a lift are applied to this body. The viscosity and thermal conductivity of the gas are neglected. The solution of Euler's equations for large distances from the body is represented in the form of three terms of an asymptotic expansion. An analogy is formulated which permits the parameters of the stream to be found from the solution of the problem of a strong "cord" blast (produced by a long highly concentrated explosive charge), when not only energy is imparted to the gas, but also a momentum is applied to it perpendicularly to the direction of the cord.

1. The form of solution for $x \to \infty$. Let us consider a stationary hypersonic flow about an arbitrary body. Let ρ_∞ be the gas density in the oncoming stream, v_∞ the gas velocity directed along the x-axis of the cylindrical system of coordinates x, r, φ. We assume that ahead of the bow shock wave the pressure $p_\infty = 0$ and consequently, the Mach number $M_\infty = \infty$. The gas is assumed to be perfect, i.e. to conform to the Clapeyron equation of state; furthermore, we take the two specific heats c_p and c_v to be constant and their ratio equal to $\varkappa$. The viscous friction stress and heat transfer in the gas are neglected. It is convenient to assume that the values of the independent variables and of the unknown functions are dimensionless and to take ρ_∞, v_∞ and r_* as the fundamental reference units; r_* is the characteristic radius of curvature of the shock front.

At some distances from the body the shape of the compression shock $r_s(x)$ is essentially determined by the wave drag. A known analogy [1 – 4] compares the hypersonic stream to the unsteady motion in a space of one less dimension. Within the framework of this analogy the velocity field associated with the drag of a body can be obtained by solving the problem of a strong explosion [5 – 9]. It follows from Sedov's law [10 – 11], established for blast waves, that for hypersonic flows $r_s \sim x^{1/2}$.

Let us assume that the flow past the body produces not only the drag but also a lift F_y. Henceforth the polar angle φ will be measured from the y-axis situated in the plane $x = \text{const}$. As in the case of plane-parallel flows [12], we assume that in the expansion for the shock front the term associated with the lift is small compared with the fundamental term depending on the drag. The form of the correction term must satisfy the obvious condition that the value of the lift must not depend on the choice of the control sections $x = \text{const}$ (ahead or behind the body) for the computation of the gas momentum component passing through these along the y-axis. As will be shown below this condition will be fulfilled if

$$r_s = (bx)^{1/2}(1 + b_y x^{-1/2} \ln x \cos\varphi + \ldots) \qquad (1.1)$$

Let us denote by v_x, v_r and v_φ the projections of the velocity vector on the axes x, r and φ. We seek the expansion for the parameters of the gas in the region behind the compression shock in the form

$$
\begin{aligned}
v_x &= 1 - \frac{1}{2(\varkappa+1)} \frac{b}{x} \{v_{x11}(\xi) + b_y x^{-1/2} [\ln x v_{x12}(\xi) + \\
&\quad v_{x13}(\xi)] \cos\varphi + \ldots\} \\
v_r &= \frac{1}{\varkappa+1} \left(\frac{b}{x}\right)^{1/2} \{v_{r11}(\xi) + b_y x^{-1/2} [\ln x v_{r12}(\xi) + v_{r13}(\xi)] \cos\varphi + \ldots\} \\
v_\varphi &= \frac{1}{\varkappa+1} \frac{b}{x} b^{-1/2} b_y \{[\ln x v_{\varphi12}(\xi) + v_{\varphi13}(\xi)] \sin\varphi + \ldots\} \\
\rho &= \frac{\varkappa+1}{\varkappa-1} \{\rho_{11}(\xi) + b_y x^{-1/2} [\ln x \rho_{12}(\xi) + \rho_{13}(\xi)] \cos\varphi + \ldots\} \\
p &= \frac{1}{2(\varkappa+1)} \frac{b}{x} \{p_{11}(\xi) + b_y x^{-1/2} [\ln x p_{12}(\xi) + p_{13}(\xi)] \cos\varphi + \ldots\} \\
\xi &= \frac{r}{(bx)^{1/2}}
\end{aligned}
\tag{1.2}
$$

Substituting formulas (1.2) in Euler's equations which we omit here for the sake of brevity, we obtain three systems of ordinary differential equations. The nonlinear system of first approximation solves the problem of a strong cord blast [10, 11]. This solution imposes the introduction of the self-similar combination ξ as one of the independent variables. The second approximation system is linear and homogeneous

$$
\begin{aligned}
&\rho_{11} \frac{dv_{r12}}{d\xi} + \left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \frac{d\rho_{12}}{d\xi} + \left(\frac{d\rho_{11}}{d\xi} + \frac{\rho_{11}}{\xi}\right) v_{r12} + \\
&\quad \left(\frac{dv_{r11}}{d\xi} + \frac{v_{r11}}{\xi} - \frac{\varkappa+1}{2}\right) \rho_{12} + \frac{\rho_{11} v_{\varphi12}}{\xi} = 0 \\
&\left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \rho_{11} \frac{dv_{r12}}{d\xi} + \frac{\varkappa-1}{2} \frac{dp_{12}}{d\xi} + \left(\frac{dv_{r11}}{d\xi} - \varkappa - 1\right) \rho_{11} v_{r12} + \\
&\quad \left[\left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \frac{dv_{r11}}{d\xi} - \frac{\varkappa+1}{2} v_{r11}\right] \rho_{12} = 0 \\
&\left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \rho_{11} \frac{dv_{\varphi12}}{d\xi} - \frac{\varkappa-1}{2} \frac{p_{12}}{\xi} + \left(\frac{v_{r11}}{\xi} - \varkappa - 1\right) \rho_{11} v_{\varphi12} = 0 \\
&\varkappa p_{11} \frac{dv_{r12}}{d\xi} + \left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \frac{dp_{12}}{d\xi} + \left(\frac{dp_{11}}{d\xi} + \varkappa \frac{p_{11}}{\xi}\right) v_{r12} + \\
&\quad \varkappa \left(\frac{dv_{r11}}{d\xi} + \frac{v_{r11}}{\xi}\right) p_{12} + \varkappa \frac{p_{11} v_{\varphi12}}{\xi} = 0 \\
&v_{x12} = \frac{1}{\varkappa+1} \left(2 v_{r11} v_{r12} + \varkappa \frac{p_{12}}{\rho_{11}} - \varkappa \frac{p_{11}\rho_{12}}{\rho_{11}^2}\right)
\end{aligned}
\tag{1.3}
$$

The third approximation system is also linear but nonhomogeneous, the functions of the first and second approximations are present in the right-hand side of the equation

$$
\begin{aligned}
&\rho_{11} \frac{dv_{r13}}{d\xi} + \left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \frac{d\rho_{13}}{d\xi} + \left(\frac{d\rho_{11}}{d\xi} + \frac{\rho_{11}}{\xi}\right) v_{r13} + \\
&\quad \left(\frac{dv_{r11}}{d\xi} + \frac{v_{r11}}{\xi} - \frac{\varkappa+1}{2}\right) \rho_{13} + \frac{\rho_{11} v_{\varphi13}}{\xi} = -(\varkappa+1)\rho_{12}
\end{aligned}
\tag{1.4}
$$

$$
\left(v_{r11} - \frac{\varkappa+1}{2}\xi\right) \rho_{11} \frac{dv_{r13}}{d\xi} + \frac{\varkappa-1}{2} \frac{dp_{13}}{d\xi} + \left(\frac{dv_{r11}}{d\xi} - \varkappa - 1\right) \rho_{11} v_{r13} +
$$

$$\left[\left(v_{r11} - \frac{\varkappa + 1}{2}\xi\right)\frac{dv_{r11}}{d\xi} - \frac{\varkappa + 1}{2}v_{r11}\right]\rho_{13} = -(\varkappa + 1)\rho_{11}v_{r12}$$

$$\left(v_{r11} - \frac{\varkappa + 1}{2}\xi\right)\rho_{11}\frac{dv_{\varphi 13}}{d\xi} - \frac{\varkappa - 1}{2}\frac{p_{13}}{\xi} + \left(\frac{v_{r11}}{\xi} - \varkappa - 1\right)\rho_{11}v_{\varphi 13} =$$
$$-(\varkappa + 1)\rho_{11}v_{\varphi 12}$$

$$\varkappa p_{11}\frac{dv_{r13}}{d\xi} + \left(v_{r11} - \frac{\varkappa + 1}{2}\xi\right)\frac{dp_{13}}{d\xi} + \left(\frac{dp_{11}}{d\xi} + \varkappa\frac{p_{11}}{\xi}\right)v_{r13} +$$
$$\varkappa\left(\frac{dv_{r11}}{d\xi} + \frac{v_{r11}}{\xi}\right)p_{13} + \varkappa\frac{p_{11}v_{\varphi 13}}{\xi} = -(\varkappa + 1)p_{12}$$

$$v_{x13} = \frac{1}{\varkappa + 1}\left(2v_{r11}v_{r13} + \varkappa\frac{p_{13}}{\rho_{11}} - \varkappa\frac{p_{11}\rho_{13}}{\rho_{11}^2}\right)$$

It is obvious that the last of equations (1.3) and (1.4) are different from the remaining and determine the perturbations v_{x12} and v_{x13} of the longitudinal component of the velocity vector, after the remaining parameters $v_{r12}, \ldots, p_{12}$ and $v_{r13}, \ldots, p_{13}$ have been found. These two groups of parameters conform to the system of equations which arise in the investigation of the second and third approximations in the theory of the two-dimensional nonstationary motion of a perfect gas. Hence, it is possible to construct the velocity field at large distances from the body and with a given accuracy ;the principle of equivalence [1 – 4] is applied here according to which the computation of the stream in an arbitrary plane $x =$ const is performed independently of the results of calculations for other planes.

The Rankine-Hugoniot conditions for the shock wave front (1.1) permit the Cauchy problem to be formulated for any of the considered systems of linear equations. We must begin their integration from the point $\xi = 1$, where, respectively,

$$v_{x12} = -dv_{x11}/d\xi, \quad v_{r12} = -dv_{r11}/d\xi, \quad v_{\varphi 12} = 1 \tag{1.5}$$
$$\rho_{12} = -d\rho_{11}/d\xi, \quad p_{12} = -dp_{11}/d\xi$$
$$v_{x13} = 4, \quad v_{r13} = 2, \quad v_{\varphi 13} = 0, \quad \rho_{13} = 0, \quad p_{13} = 4 \tag{1.6}$$

2. Calculation of the lift. By a direct check it is possible to ascertain that the solution of the system (1.5) that satisfies the initial conditions (1.5) has the form

$$v_{x12} = -dv_{x11}/d\xi, \quad v_{r12} = -dv_{r11}/d\xi, \quad v_{\varphi 12} = v_{r11}/\xi \tag{2.1}$$
$$\rho_{12} = -d\rho_{11}/d\xi, \quad p_{12} = -dp_{11}/d\xi$$

The existence of this simple solution is due to the group properties of the initial Euler's equations. They are invariant, particularly with respect to the displacement along all three axes of the Cartesian system of coordinates. Let us replace $r = \sqrt{y^2 + z^2}$ by $r = \sqrt{(y + \Delta y)^2 + z^2}$ in Sedov's solution for a strong cord blast and assume that $\Delta y \ll y$. We expand the relations obtained in this way in series and restrict ourselves to terms of the first order with respect to Δy. Their coefficients are proportional to $x^{-1/2}$ and contain the functions of the self-similar variable ξ which are given by the formulas (2.1). Hence, the reason for introducing the logarithmic terms into the asymptotic representations (1.1) and (1.2) for the compression shock and for the parameters of gas behind the shock, becomes obvious. If we omit these terms and set the value $b_y x^{-1/2}\cos\varphi$ instead of the points in the right-hand side of (1.1), then the perturbations in the basic solution will be of the order of $x^{-1/3}$ for $x \to \infty$. The right-hand sides of Eqs. (1.4) vanish and the initial data for the functions $v_{x13}, \ldots, p_{13}$ will be determined by the relations (1.5) and not by (1.6). As a result, the formulas (2.1) will give the solution of the equations for the third approximation. It follows from the reasons given above that in this case the sum of the fundamental and the correction terms in the expansion (1.2) represents an approximate solution of the problem of strong blast with the symmetry axis shifted in the plane xy parallel to the axis $r = 0$ in the cylindrical system of coordinates. The possibility of such a shift permits the term proportional to $x^{-1/3}$ to be eliminated from the asymptotic representation (1.1) of the shock front.

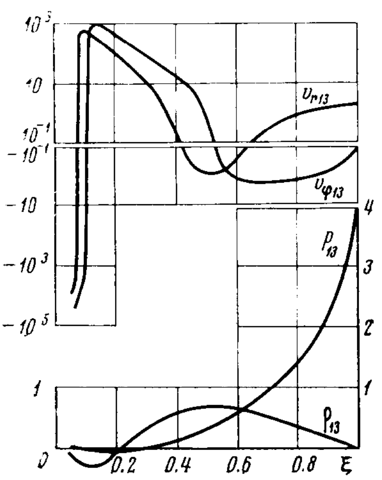

Fig. 1

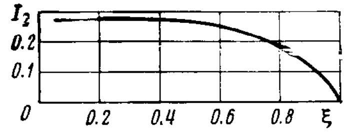

Fig. 2

Let us consider the system of equations (1.1) for the functions of the third approximation. As noted above, the front of the expansion (1.2) was chosen in order to obtain a finite value for the lift by calculating the y-component of the gas momentum transferred through a closed control surface placed at a sufficiently large distance from the body. This allows us to write the integral of the considered system [13]. Taking into account the formulas (2.1), we have

$$\left(2v_{r11} - \frac{\varkappa+1}{2}\xi\right)\rho_{11}v_{r13} - \left(v_{r11} - \frac{\varkappa+1}{2}\xi\right)\rho_{11}v_{\varphi13} + \left(v_{r11} - \frac{\varkappa+1}{2}\xi\right)v_{r11}\rho_{13} + \frac{\varkappa-1}{2}p_{13} = \frac{C}{\xi} + (\varkappa+1)v_{r11}\rho_{11} \tag{2.2}$$

To calculate the constant C we use the initial data (1.6) and as a result we obtain $C = 0$. Let us now represent the function $v_{\varphi13}$ by the remaining unknown quantities using the integral (2.2). In the system (1.4) one of the first three equations is dependent on the other two, therefore it can be omitted. It is convenient to integrate the system numerically. It was assumed in the calculations that the Poisson's adiabatic exponent $\varkappa = 1.4$. The curves of the components v_{r13} and $v_{\varphi13}$ of the perturbed velocity vector, of the excess density ρ_{13} and the excess pressure p_{13} are given in Fig. 1. For $\xi \to 0$ all these functions have an oscillatory character, but as v_{r13} and $v_{\varphi13}$ increase infinitely, ρ_{13} and p_{13} tend to zero.

We set two control planes normal to the direction of the oncoming stream. Let one of them be placed in the stream in front of the body, and the other behind it at a distance x. We write the expression for the lift

$$F_y = -\lim_{r\to 0}\int_r^{r_s}\int_0^{2\pi} \rho v_x v_y r\,dr\,d\varphi$$

Substituting here the expansions (1.2) and retaining all terms up to and including that of the order of x°, we have

$$F_y = -\frac{\pi}{\varkappa-1} b^{3/2} b_y [(1+I_1)\ln x + \lim_{\xi\to 0} I_2(\xi)]$$

$$I_1 = \int_0^1 (\rho_{11}v_{r12} + v_{r11}\rho_{12} - \rho_{11}v_{\varphi 12})\,\xi d\xi$$

$$I_2(\xi) = \int_\xi^1 (\rho_{11}v_{r13} + v_{r11}\rho_{13} - \rho_{11}v_{\varphi 13})\,\xi d\xi$$

It is easy to compute the integral I_1 bearing in mind the solution (2.1) for the functions of the second approximation. In fact,

$$I_1 = -\int_0^1 \frac{d}{d\xi}(v_{r11}\rho_{11}\xi)\,d\xi = -1$$

As a result we obtain the following expression for the lift

$$F_y = -\frac{\pi}{\varkappa-1} b^{3/2} b_y \lim_{\xi\to 0} I_2(\xi) \tag{2.3}$$

The behavior of the integral $I_2(\xi)$ can be seen in Fig. 2. In spite of the infinite increase of the components v_{r13} and $v_{\varphi 13}$ of the perturbed velocity vector for $\xi \to 0$, this integral tends to a finite value $I_2(0) = 0.2775$. A rigorous proof of the convergence of $I_2(\xi)$ will be given in Sect. 3, following the analysis of the asymptotic behavior of the third approximation functions.

Let us consider the dependence of the solution we have constructed on the angular variable. The addition to the correction terms in the expansions (1.1) and (1.2) of the terms containing $\sin\varphi$, results in the appearance of a lateral force F_z which (beside the lift) acts on the body in the plane normal to the direction of the oncoming stream. The computation of F_z is exactly analogous to that of F_y. The additional correction terms containing higher harmonics $\cos k\varphi$ and $\sin k\varphi$ $(k = 2,3, \ldots)$ can also be chosen proportional to $x^{-1/2}$, but they do not contribute to the forces. For these reasons in the Fourier series which represent the dependence of the gas parametrs on the angular variable, only the terms with $\cos\varphi$ were retained right from the beginning.

As was mentioned above, the functions $v_{r12}, \ldots, p_{12}$ and $v_{r13}, \ldots, p_{13}$ are determined by the system of equations which obtain in the analysis of higher approximations in the theory of the two-dimensional motion of a perfect gas, and are a little different from the self-similar ones. It follows from the results of [13] that the integral (2.2) exists only in the case when the expression for the momentum of the matter inside the perturbed region of nonstationary flow, contains components which do not vary with time. In a stationary hypersonic stream the time is represented by the longitudinal coordinate x and the momentum is represented by the lift F_y. Hence, it is obvious that in each plane $x = \text{const}$ the parameters of the hypersonic stream for an arbitrary body which is under the action of drag and lift, can be found from the solution of the problem of a strong cord blast when not only energy is transferred to the gas, but also a momentum normal to the cord direction (along the y-axis). The analogy with a blast, which shows the relation between the energy emitted per unit of the cord length and the drag of the body, has been known for a long time [5 – 9]. Using the formulation given above, this analogy permits us to compare the momentum per unit length of the blast wave and the lift applied to the body in a hypersonic stream.

3. Asymptotics of the functions of the third approximation for $\xi \to 0$. The behavior of the second approximation functions for $\xi \to 0$ can be determined by formulas (2.1), substituting into their right-hand sides the known asymptotic expansion of the solution of the problem of a strong cord blast [14]. Let us clarify the behavior of the functions of the third approximation. To do this we revert to the system of equations (1.4), without first using the simplifications related to the existence of the integral (2.2). As a result, for the first linearly independent solution of the homogeneous system corresponding to (1.4), we have

$$
\begin{aligned}
&v_{x13} = -c_1 \frac{\varkappa}{\varkappa+1} \frac{k_2}{k_1^2} \xi^{-(3\varkappa+1)/(\varkappa-1)} + \ldots \\
&v_{r13} = c_1 \frac{(1-2\varkappa)(\varkappa^2-1)}{4\varkappa k_1} \xi^{-2\varkappa/(\varkappa-1)} + \ldots \\
&v_{\varphi 13} = c_1 \frac{(1-2\varkappa)(\varkappa+1)^2}{4\varkappa k_1} \xi^{-2\varkappa/(\varkappa-1)} + \ldots \\
&\rho_{13} = c_1 \xi^{-3} + \ldots, \qquad p_{13} = c_1 \frac{(1-2\varkappa)(\varkappa+1)^3}{4\varkappa^2(\varkappa-1)} \xi^{-1} + \ldots
\end{aligned}
\tag{3.1}
$$

The second of the linearly independent solutions is taken in the form

$$
\begin{aligned}
&v_{x13} = -c_2 \frac{\varkappa}{\varkappa+1} \frac{k_2}{k_1^2} \xi^{-(2\varkappa+1)/(\varkappa-1)} \cos(k \ln \xi) + \ldots \\
&v_{r13} = -c_2 \frac{(\varkappa+1)(\varkappa-1)^2}{4\varkappa k_1} \xi^{-\varkappa/(\varkappa-1)} [\cos(k \ln \xi) + k \sin(k \ln \xi)] + \ldots \\
&v_{\varphi 13} = c_2 \frac{(\varkappa+1)(\varkappa-1)^2}{4\varkappa k_1} \xi^{-\varkappa/(\varkappa-1)} \left[\frac{2-\varkappa}{\varkappa-1} \cos(k \ln \xi) - \right. \\
&\qquad \left. \frac{\varkappa k}{\varkappa-1} \sin(k \ln \xi)\right] + \ldots \\
&\rho_{13} = c_2 \xi^{(3-2\varkappa)/(\varkappa-1)} \cos(k \ln \xi) + \ldots \\
&p_{13} = c_2 \frac{(\varkappa+1)^2(\varkappa-1)}{2\varkappa^2} \xi^{1/(\varkappa-1)} \left[\frac{1}{\varkappa-1} \cos(k \ln \xi) + k \sin(k \ln \xi)\right] + \ldots
\end{aligned}
\tag{3.2}
$$

We write the third linearly independent solution in the form

$$
\begin{aligned}
&v_{x13} = -c_3 \frac{\varkappa}{\varkappa+1} \frac{k_2}{k_1^2} \xi^{-(2\varkappa+1)/(\varkappa-1)} \sin(k \ln \xi) + \ldots \\
&v_{r13} = c_3 \frac{(\varkappa+1)(\varkappa-1)^2}{4\varkappa k_1} \xi^{-\varkappa/(\varkappa-1)} [k \cos(k \ln \xi) - \sin(k \ln \xi)] + \ldots \\
&v_{\varphi 13} = c_3 \frac{(\varkappa+1)(\varkappa-1)^2}{4\varkappa k_1} \xi^{-\varkappa/(\varkappa-1)} \left[\frac{\varkappa k}{\varkappa-1} \cos(k \ln \xi) + \right. \\
&\qquad \left. \frac{2-\varkappa}{\varkappa-1} \sin(k \ln \xi)\right] + \ldots \\
&\rho_{13} = c_3 \xi^{(3-2\varkappa)/(\varkappa-1)} \sin(k \ln \xi) + \ldots
\end{aligned}
\tag{3.3}
$$

$$p_{13} = -c_3 \frac{(\varkappa+1)^2(\varkappa-1)}{2\varkappa^2} \xi^{1/(\varkappa-1)} \Big[k\cos(k\ln\xi) - \frac{1}{\varkappa-1}\sin(k\ln\xi)\Big] + \ldots$$
$$k = \sqrt{(3-\varkappa)/(\varkappa-1)}$$

Here k_1 and k_2 are the coefficients of the leading terms in the expansions of the functions ρ_{11} and p_{11} from the problem on a strong explosion; their values can be found in the monograph of Sedov [14]. The fourth linearly independent solution of a homogeneous and the partial solution of a nonhomogeneous system of Eqs. (1.4) bring some contribution only to the lower order terms in the asymptotic representation of the functions $v_{x13}, \ldots, p_{13}$ for $\xi \to 0$; therefore they are not quoted.

If the constant $c_1 \neq 0$, then the estimate for the integrand in the right-hand side of the formula (2.3) is given by

$$(\rho_{11}v_{r13} + v_{r11}\rho_{13} - \rho_{11}v_{\varphi 13})\xi \sim c_1\xi^{-1}$$

Hence, it is clear that for $\xi \to 0$ for the convergence of the integral $I_2(\xi)$ to the finite limit $I_2(0)$, it is necessary to satisfy the condition $c_1 = 0$. Now we shall use the equality (2.2) which defines the relation between the functions of the third approximation. As follows from simple computations $c_1 = C$, but the constant $C = 0$ according to Cauchy's data (1.6). Lowering the order of the system of equations (1.4) from the fourth to the third which was obtained by excluding the quantity $v_{\varphi 13}$ from it, permitted in Sect. 2 the solution resulting in the disappearance of the asymptotics (3.1) to be at once constructed. Since the asymptotics (3.2) and (3.3) which represent subsequent terms of the expansion ensure the convergence of the integral $I_2(\xi)$, the computations yielded the final value $I_2(0) = 0.2775$. Thus, for $\varkappa = 1.4$ the lift of the body

$$F_y = -0.6241\ b^{3/2}b_y$$

As for the constants c_2 and c_3, numerical integration of the system of equations (1.4) after the elimination of function $v_{\varphi 13}$ yields $c_2 = 0.085$ and $c_3 = -0.978$.

Asymptotic expansions confirm that for $\xi \to 0$ all the third order functions have an oscillatory character. The period $L = (e^{2\pi/k} - 1)\xi$ vanishes with the self-similar variable ξ, i.e. the oscillation frequency ω increases as the axis $r = 0$ is approached. The amplitude of oscillations of the components of the perturbed velocity vector is determined by the behavior of the quantities v_{x13}, v_{r13} and $v_{\varphi 13}$; it increases infinitely, while the amplitude of oscillations of the excess pressure p_{13} decreases to zero. Oscillations of the excess density ρ_{13} vary with the Poisson's adiabatic exponent; for $\varkappa < 1{,}5$ their amplitude diminishes and for $\varkappa > 1{,}5$ increases infinitely.

For $r = \text{const}$ and $x \to \infty$ the self-similar variable $\xi \to 0$. Hence, the parameters of the gas oscillate not only near the axis $r = 0$ but also downstream at any fixed distance from the axis. In this case the oscillation frequency $\omega \sim x^{1/2}$.

REFERENCES

1. Tsien, H. S., Similarity laws of hypersonic flows. J. Math., Phys., Vol. 25, № 3, 1946.

2. Hayes, W. D., On hypersonic similitude. Quart. Appl. Math., Vol. 5, № 1, 1947.

3. Il'iushin, A. A., The law of plane sections in the aerodynamics of high supersonic speeds. PMM Vol. 20, № 6, 1956.

4. Bam-Zelikovich, G. M., Bunimovich, A. I. and Mikhailova, M. P., The motion of slender bodies at high supersonic speeds. Izv. Akad. Nauk SSSR, OTN, Mekhanika i Mashinostroenie, № 1, 1960.

5. Cheng, H. K. and Pallone, A. J., Inviscid leading-edge effect in hypersonic flow. J. Aeronaut. Sci., Vol. 23, № 7, 1956.

6. Lees, L. and Kubota, T., Inviscid hypersonic flow over blint-nosed slender bodies. Aeronaut. J. Sci., Vol. 24, № 3, 1957.

7. Chernyi, G. G., The effect of slight blunting of the leading edge of a profile on the high supersonic flow past it. Dokl. Akad. Nauk SSSR, Vol. 114, № 4, 1957.

8. Chernyi, G. G., The flow past a blunted slender cone at high supersonic speed. Dokl. Akad. Nauk SSSR, Vol. 115, № 4, 1957.

9. Chernyi, G. G., The effect of slight blunting of the leading end of a body at the high supersonic speed flow past it. Izv. Akad. Nauk SSSR, OTN, № 4, 1958.

10. Sedov, L. I., The motion of air in strong explosions. Dokl. Akad. Nauk SSSR, Vol. 52, № 1, 1946.

11. Sedov, L. I., Propagation of strong blast waves. PMM Vol. 10, № 2, 1946.

12. Ryzhov, O. S. and Terent'ev, E. D., On the hypersonic flow past a lift airfoil. PMM Vol. 38, № 1, 1974.

13. Ryzhov, O. S. and Terent'ev, E. D., On the general theory of almost self-similar nonstationary flows. PMM Vol. 37, № 1, 1973.

14. Sedov, L. I., Similarity and Dimensional Anaysis Methods in Mechanics. Moscow, "Nauka", 1967.

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